

Muon Anomalous Magnetic Moment and Lepton Flavor Violating Tau Decay in Unparticle Physics

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Abstract

We study effects of unparticle physics on muon $g - 2$ and LFV tau decay processes. LFV interactions between the Standard Model sector and unparticles can explain the difference of experimental value of muon $g - 2$ from the Standard Model prediction. While the same couplings generate LFV tau decay, we found that LFV coupling can be of $\mathcal{O}(0.1 \dots 1)$ without conflict with experimental bounds of LFV tau decay if the scaling dimension of unparticle operator $d_{\mathcal{U}} \gtrsim 1.6$.

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I. INTRODUCTION

Scale or conformal invariance of a quantum field theory requires that particles included in that theory be massless. In the Standard Model (SM), scale invariance is broken by the mass parameter in the Higgs sector and by running of gauge couplings. However, this does not forbid the existence of scale invariant hidden sector. Recently, motivated by Banks and Zaks [1], Georgi suggested [2] that there may exist a scale invariant hidden sector of unparticles \mathcal{U} coupled to the Standard Model (SM) at TeV scale. The theory at high energy contains both the SM fields and so-called Banks-Zaks (\mathcal{BZ}) fields of a theory with a non-trivial infrared fixed point, interacting via messenger fields of high mass. At the TeV scale, \mathcal{BZ} fields are mapped to effective scale invariant unparticle operators interacting with the SM fields. An intriguing property of unparticles is their non-integral scaling dimension $d_{\mathcal{U}}$. They behave like $d_{\mathcal{U}}$ number of massless invisible particles.

Since the principle to constrain the interactions of unparticles with SM sector is still unknown, there are many possibilities of interactions that preserve Lorentz structure: unparticles that are SM gauge singlets [3], have baryon number [4] or gauge quantum numbers [5]. Moreover, Lepton Flavor Violating (LFV) as well as Conserving (LFC) interactions are possible. The unparticle physics based on these interactions has rich phenomenological implications, and it has been studied by many authors for collider signature of unparticles [6], neutral meson mixing system [7], muon anomalous magnetic moment ($g - 2$) [8, 9, 10, 11, 12], LFV processes [13, 14, 15, 16, 17, 18, 19], and so on.

Experimental results [20] for muon anomalous magnetic moment $a_{\mu} = (g - 2)_{\mu}/2$ are a promising hint of new physics beyond the SM. It is well known that between the experimental value and the SM prediction there is difference [21, 22]

$$\Delta a_{\mu} = a_{\mu}^{\text{exp}} - a_{\mu}^{\text{SM}} = 29.5(8.8) \times 10^{-10}, \quad (1)$$

with a discrepancy of 3.4σ . There have been many attempts to explain this discrepancy by new physics (see [22, 23, 24] and references therein).

In this paper, we investigate the muon $g - 2$ and LFV tau decays mediated by scalar unparticles. If there are LFC and LFV couplings of unparticles with charged leptons, these couplings contribute to $g - 2$ at one-loop level by muon loop from LFC $\mu\mu\mathcal{U}$ coupling [8, 9, 10, 11, 12] and tau loop from LFV $\tau\mu\mathcal{U}$ coupling. These couplings generate LFV tau decay processes $\tau \rightarrow 3\mu$ at tree level and $\tau \rightarrow \mu\gamma$ at one-loop level as well. If the discrepancy Eq. (1) is saturated by unparticles, one can constrain the coupling constants and the scaling dimension $d_{\mathcal{U}}$ without conflicting the experimental bound on LFV tau decays.

This paper is organized as follows: in Section II, we give a brief introduction of unparticles. In Section III, unparticle mediated muon $g - 2$ is studied. Section IV is devoted to LFV tau decay. We find that there exists a consistent region of the coupling constants and scaling dimension $d_{\mathcal{U}}$ that is compatible with both muon $g - 2$ and LFV tau decay processes. We conclude in Section V.

II. UNPARTICLES

Unparticles are scale invariant objects originating from a hidden \mathcal{BZ} sector with a non-trivial infrared fixed point. This \mathcal{BZ} sector is assumed to interact with the SM sector by exchanging very heavy particles at a high scale $M_{\mathcal{U}}$. The effective operator of that interaction has the form

$$\frac{1}{M_{\mathcal{U}}^{d_{\text{SM}}+d_{\mathcal{BZ}}-4}} \mathcal{O}_{\text{SM}} \mathcal{O}_{\mathcal{BZ}}, \quad (2)$$

where $\mathcal{O}_{\text{SM}(\mathcal{BZ})}$ is an operator constructed by fields of the SM (\mathcal{BZ}) sector with mass dimension $d_{\text{SM}(\mathcal{BZ})}$. Renormalization effects in the \mathcal{BZ} sector induce dimensional transmutation at the scale $\Lambda_{\mathcal{U}} \sim 1$ TeV. Below this scale, \mathcal{BZ} fields match onto unparticle operators $\mathcal{O}_{\mathcal{U}}$, and effective interactions with the SM sector are written as

$$\frac{C_{\mathcal{U}} \Lambda_{\mathcal{U}}^{d_{\mathcal{BZ}}-d_{\mathcal{U}}}}{M_{\mathcal{U}}^{d_{\text{SM}}+d_{\mathcal{BZ}}-4}} \mathcal{O}_{\text{SM}} \mathcal{O}_{\mathcal{U}} = \frac{\lambda}{\Lambda_{\mathcal{U}}^{d_{\text{SM}}+d_{\mathcal{U}}-4}} \mathcal{O}_{\text{SM}} \mathcal{O}_{\mathcal{U}}, \quad (3)$$

where $C_{\mathcal{U}}$ is a coefficient fixed by the matching condition and $\lambda = C_{\mathcal{U}}(\Lambda_{\mathcal{U}}/M_{\mathcal{U}})^{d_{\text{SM}}+d_{\mathcal{BZ}}-4}$. Although in principle the form of unparticle operator $\mathcal{O}_{\mathcal{U}}$ is determined by the theory in the hidden sector, the latter is yet unknown, and only Lorentz invariance constrains the unparticle operators.

In this paper we consider LFV interactions between SM fields and unparticles of scalar (S) and pseudo-scalar (P) type.

$$\mathcal{L} = \frac{\lambda_{ij}^S}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}-1}} \bar{\ell}_i \ell_j \mathcal{O}_{\mathcal{U}} + \frac{\lambda_{ij}^P}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}-1}} \bar{\ell}_i i \gamma^5 \ell_j \mathcal{O}_{\mathcal{U}}, \quad (4)$$

where $\ell_i (i = e, \mu, \tau)$ denotes charged lepton of i th generation, and we assume unparticle scale $\Lambda_{\mathcal{U}} = 1$ TeV throughout this paper. The coupling constant λ_{ij}^S and λ_{ij}^P are assumed to be real.

Propagators of scalar unparticle of momentum P is derived from the principle of scale invariance as [2, 8]

$$\frac{i A_{d_{\mathcal{U}}}}{2 \sin d_{\mathcal{U}} \pi} (-P^2 - i\epsilon)^{d_{\mathcal{U}}-2}, \quad (5)$$

where the normalization factor $A_{d_{\mathcal{U}}}$ is

$$A_{d_{\mathcal{U}}} = \frac{16\pi^{5/2}}{(2\pi)^{2d_{\mathcal{U}}}} \frac{\Gamma(d_{\mathcal{U}} + 1/2)}{\Gamma(d_{\mathcal{U}} - 1)\Gamma(2d_{\mathcal{U}})}. \quad (6)$$

In this paper, we consider only the region $1 < d_{\mathcal{U}} < 2$ which comes from unitarity condition ($1 < d_{\mathcal{U}}$) [2, 25, 26] and convergence condition ($d_{\mathcal{U}} < 2$).

III. MUON ANOMALOUS MAGNETIC MOMENT

In this section we consider unparticle contributions to muon $g - 2$ by both LFC and LFV interactions given in Eq. (4). If we assume that unparticles explain the difference Δa_μ of between experimentally measured muon $g - 2$ and its SM prediction Eq. (1), this condition restricts the possible range of the couplings. New contributions to Δa_μ by (pseudo)scalar unparticles are generated by one-loop diagram (Fig. 1), and the results are

$$\Delta a_\mu^S = - \sum_{j=e,\mu,\tau} \frac{|\lambda_{\mu j}^S|^2}{8\pi^2} \left(\frac{m_j^2}{\Lambda_\mathcal{U}^2} \right)^{d_\mathcal{U}-1} \mathcal{Z}_{d_\mathcal{U}} \sqrt{r_j} \int_0^1 dz F^S(z, d_\mathcal{U}, r_j), \quad (7)$$

$$\Delta a_\mu^P = + \sum_{j=e,\mu,\tau} \frac{|\lambda_{\mu j}^P|^2}{8\pi^2} \left(\frac{m_j^2}{\Lambda_\mathcal{U}^2} \right)^{d_\mathcal{U}-1} \mathcal{Z}_{d_\mathcal{U}} \sqrt{r_j} \int_0^1 dz F^P(z, d_\mathcal{U}, r_j), \quad (8)$$

where $j = (e, \mu, \tau)$ denotes the flavor of internal charged leptons, $r_j = m_\mu^2/m_j^2$, $\mathcal{Z}_{d_\mathcal{U}} = A_{d_\mathcal{U}}/(2 \sin d_\mathcal{U} \pi)$ and functions under Feynman parameter integrals are defined as

$$F^S(z, d_\mathcal{U}, r_j) = z^{1-d_\mathcal{U}} (1-z)^{d_\mathcal{U}} (1 + \sqrt{r_j} z) (1 - r_j z)^{d_\mathcal{U}-2}, \quad (9)$$

$$F^P(z, d_\mathcal{U}, r_j) = z^{1-d_\mathcal{U}} (1-z)^{d_\mathcal{U}} (1 - \sqrt{r_j} z) (1 - r_j z)^{d_\mathcal{U}-2}. \quad (10)$$

Contribution from pseudoscalar interactions Δa_μ^P is obtained by replacing m_j with $-m_j$ in Δa_μ^S from the chirality structure. One can easily verify that Eqs. (7)–(8) reduce to the formulae of [9] in the case of flavor-blind interactions when $r_j = 1$.

The contribution from scalar interactions Δa_μ^S to Eq. (1) is positive for all $j = e, \mu, \tau$. The contribution from pseudoscalar interactions Δa_μ^P has the same sign as Δa_μ^S for $j = e$ because of $r_{j=e} \gg 1$. However, for $j = \mu, \tau$ the contribution of Δa_μ^P to muon $g - 2$ is negative. Therefore, we assume that pseudo-scalar couplings $\lambda_{\mu j}^P = 0$, ($j = \mu, \tau$) in the following analysis. The treatment of this (μ, e) LFV coupling is discussed below.

Fig. 2 shows Δa_μ as a function of $d_\mathcal{U}$ calculated from Eq. (7) with various values of $\lambda_{\mu\tau, \mu\mu, \mu e}^S$ with $\Lambda_\mathcal{U} = 1$ TeV. The solid, thick-solid, dashed, thick-dashed and dotted curve correspond to $(\lambda_{\mu\tau}^S, \lambda_{\mu\mu}^S, \lambda_{\mu e}^S) = (1, 0, 0), (10^{-4}, 0, 0), (0, 1, 0), (0, 10^{-4}, 0)$ and $(0, 0, 1)$, respectively. The horizontal lines represent upper and lower value of Eq. (1). From finite values of Δa_μ at $d_\mathcal{U} = 1$, they are decreasing with larger $d_\mathcal{U}$, but diverge at $d_\mathcal{U} = 2$, while the dotted curve for $d_\mathcal{U} < 1.5$ is not plotted because it becomes negative in that region. Contribution of $\lambda_{\mu e}^S$ (dotted curve) is negative for $d_\mathcal{U} < 1.5$ and below the experimental value for almost all region of $d_\mathcal{U} > 1.5$ even if $\lambda_{\mu e}^S = 1$. Moreover, this (μ, e) LFV coupling must be suppressed by experiments of $\mu \rightarrow e\gamma$ and $\mu - e$ conversion in nuclei [14], and $\mu \rightarrow 3e$ decay process [15] ($\lambda_{\mu e}^S = 0$ is consistent with these experiments). In fact, at $d_\mathcal{U} = 1.55$ which gives the largest contribution to Δa_μ , upper bound of $\lambda_{\mu e}^S$ is 10^{-2} from $\mu \rightarrow e\gamma$ and 10^{-4} from $\mu \rightarrow 3e$ for $\lambda_{\mu\mu} = \lambda_{ee} = 10^{-4}$. Such small (or vanishing) (μ, e) LFV coupling can not play significant role here. Therefore, we neglect this coupling in the following analysis.

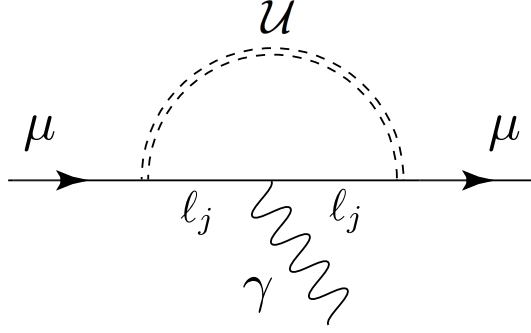


FIG. 1: One-loop diagram of muon $g - 2$ mediated by unparticles \mathcal{U} .

Fig. 3 shows the consistent region of $\lambda_{\mu\tau}^S$ (left) and $\lambda_{\mu\mu}^S$ (right) at $\Lambda_{\mathcal{U}} = 1$ TeV under the assumption that both LFC and LFV couplings simultaneously contribute to Δa_μ , and it is in the bound of Eq. (1). For $\lambda_{\mu\tau}^S$, $\lambda_{\mu\mu}^S$ is only a free parameter, and *vice versa*. For both couplings, λ^S s have to be small for the region of small $d_{\mathcal{U}}$, but they can be of $\mathcal{O}(1)$ for relatively large $d_{\mathcal{U}}$. They must be extremely small when $d_{\mathcal{U}}$ is closer to 2, and there is no solution at $d_{\mathcal{U}} = 2$ because Δa_μ diverges at this point. Since we have assumed that both $\lambda_{\mu\tau}^S$ and $\lambda_{\mu\mu}^S$ contribute, these allowed regions are not independent of each other. The relation between these two couplings are shown in Fig. 4. These figures represent the allowed region of λ^S s in the $\lambda_{\mu\tau}^S - \lambda_{\mu\mu}^S$ plane with $d_{\mathcal{U}} = (1.7, 1.9)$ (left) and $d_{\mathcal{U}} = (1.6, 1.8)$ (right). In the left panel, the region surrounded by thick-solid (solid) curves correspond to $d_{\mathcal{U}} = 1.7(1.9)$, and similarly for the right panel. The “slice” of this area goes inside with decreasing $d_{\mathcal{U}}$, and we emphasize that both or at least one coupling of $\lambda_{\mu\tau(\mu\mu)}^S$ have to be of $\mathcal{O}(0.1 \dots 1)$ for large $d_{\mathcal{U}}$ ($\gtrsim 1.6$). These relatively large couplings may raise a problem on LFV tau decay that we study in the next section.

IV. LFV TAU DECAY

In this section, we investigate the LFV τ decay processes generated by the same unparticle interactions as those of Δa_μ . We discuss $\tau \rightarrow 3\mu$ and $\tau \rightarrow \mu\gamma$. Since the constraints of couplings $\lambda_{\mu\tau(\mu\mu)}^S$ obtained from the consistency of Δa_μ in the previous section tolerate large LFV couplings, our next task is to make certain that this does not conflict with the experimental bound of LFV tau decays [21, 27]

$$BR(\tau \rightarrow 3\mu) < 3.2 \times 10^{-8}, \quad BR(\tau \rightarrow \mu\gamma) < 6.8 \times 10^{-8}. \quad (11)$$

Here, we will find regions of couplings which are consistent with both Δa_μ and LFV tau decays.

First we consider $\tau \rightarrow 3\mu$ LFV tau decay [15, 16]. This decay mode mediated by unparticle operators occurs at tree-level (Fig. 5, left).

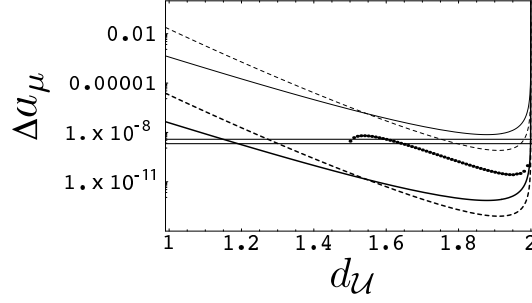


FIG. 2: Δa_μ from the couplings with scalar unparticles as a function of the scaling dimension d_U with $\Lambda_U = 1$ TeV. The solid, thick-solid, dashed, thick-dashed and dotted curves correspond to $(\lambda_{\mu\tau}^S, \lambda_{\mu\mu}^S, \lambda_{\mu e}^S) = (1, 0, 0), (10^{-4}, 0, 0), (0, 1, 0), (0, 10^{-4}, 0)$ and $(0, 0, 1)$, respectively. All curves diverge at $d_U = 2$. The horizontal lines are the upper and lower value of Eq. (1).

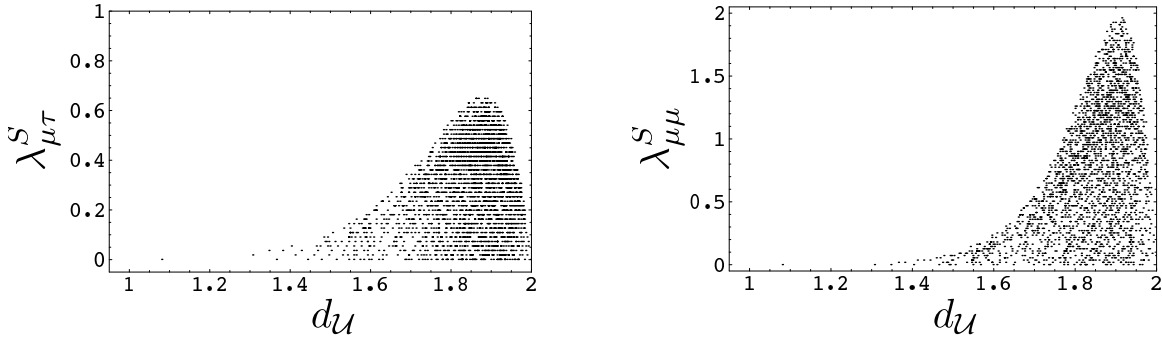


FIG. 3: Allowed region of $\lambda_{\mu\tau}^S$ (left) and $\lambda_{\mu\mu}^S$ (right) from the condition of Δa_μ .

The decay rate of $\tau \rightarrow 3\mu$ derived in [15] is

$$\frac{d\Gamma}{ds \sin \theta d\theta} = \frac{1}{2^9 \pi^3} \frac{1}{\sqrt{s}} \sqrt{\left(1 - \frac{(m_\tau - m_\mu)^2}{s}\right) \left(1 - \frac{(m_\tau + m_\mu)^2}{s}\right)} \sqrt{1 - \frac{4m_\mu^2}{s}} \sum_{\text{spin}} |\mathcal{M}|^2, \quad (12)$$

where θ is angle between three-momenta \mathbf{p}_1 and \mathbf{p}_4 , $s = (p_1 - p_2)^2$ and its integral range is $4m_\mu^2 \leq s \leq (m_\tau - m_\mu)^2$. Amplitude \mathcal{M} is

$$\begin{aligned} \sum_{\text{spin}} |\mathcal{M}|^2 = & 4 |\lambda_{\mu\tau}^S|^2 |\lambda_{\mu\mu}^S|^2 [4(p_1 \cdot p_2)(p_3 \cdot p_4)|F_1|^2 + 4(p_1 \cdot p_3)(p_2 \cdot p_4)|F_2|^2 \\ & - 2 \{(p_1 \cdot p_2)(p_3 \cdot p_4) + (p_1 \cdot p_3)(p_2 \cdot p_4) - (p_1 \cdot p_4)(p_2 \cdot p_3)\} \text{Re}(F_1 F_2^*)], \end{aligned} \quad (13)$$

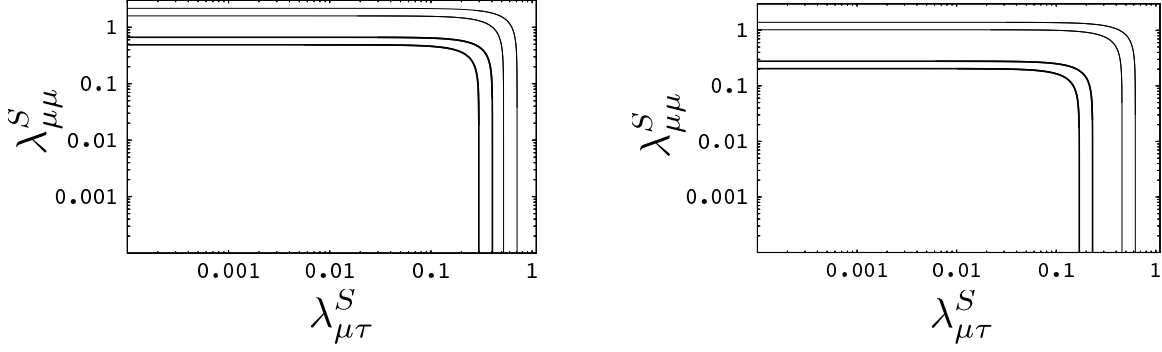


FIG. 4: Allowed region of the coupling constants in the $\lambda_{\mu\tau}^S - \lambda_{\mu\mu}^S$ plane with $d_U = (1.7, 1.9)$ (left) and $d_U = (1.6, 1.8)$ (right). In the left panel, the region surrounded by thick-solid (solid) curves corresponds to $d_U = 1.7(1.9)$, and similarly for the right panel with $d_U = 1.6(1.8)$.

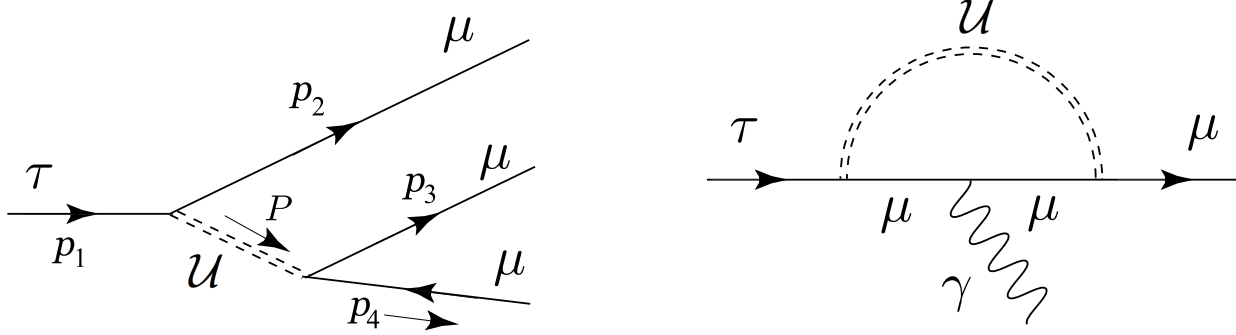


FIG. 5: Tree-level diagram of LFV $\tau \rightarrow 3\mu$ decay mediated by unparticles (left) and $\tau \rightarrow \mu\gamma$ at one-loop level (right). For $\tau \rightarrow 3\mu$, there also exists the u -channel diagram by exchanging external muons of momenta p_2 and p_3 .

where

$$\begin{aligned} F_1 &= \frac{\mathcal{Z}_{d_U}}{\Lambda_U^{2(d_U-1)}} (-(p_1 - p_2)^2 - i\epsilon)^{d_U-2}, \\ F_2 &= \frac{\mathcal{Z}_{d_U}}{\Lambda_U^{2(d_U-1)}} (-(p_1 - p_3)^2 - i\epsilon)^{d_U-2}. \end{aligned} \quad (14)$$

LFV unparticle interactions can generate other $\tau \rightarrow 3\ell$ decay processes in general, and $\tau \rightarrow e\mu\mu$ is the one of them which contains the coupling responsible for muon $g-2$. This process contains $\lambda_{\mu e}^S$ as well as $\lambda_{\mu\tau}^S$. However, as mentioned in the previous section, we have neglected this (μ, e) LFV coupling because it is strongly suppressed by other LFV processes.

$\tau \rightarrow \mu\gamma$ process is the other decay mode by the same unparticle interactions at one-loop level

(Fig. 5, right). Decay Rate of this process is [14]

$$\Gamma = \frac{m_\tau^3}{8\pi} |\mathcal{A}|^2 \quad (15)$$

where the amplitude \mathcal{A} is given by

$$\mathcal{A} = - \sum_{j=e,\mu,\tau} \frac{ie}{(4\pi)^2} \lambda_{\mu j}^S \lambda_{j\tau}^S \mathcal{Z}_{d_U} \frac{1}{(\Lambda_U^2)^{d_U-1}} G_j^S(d_U), \quad (16)$$

and functions $G_j^S(d_U)$ are

$$G_j^S(d_U) = \int dx dy dz \delta(x+y+z-1) z^{1-d_U} [-xzm_\tau^2 - yzm_\mu^2 + (x+y)m_j^2]^{d_U-2} [xzm_\tau + yzm_\mu + (x+y)m_j]. \quad (17)$$

While three leptons $j = (e, \mu, \tau)$ can exist in the loop, we consider only the case of muon virtual particle because we are interested in tau decays generated by the same couplings as those of muon $g-2$. Contributions from other virtual particles depend on different combination of λ^S , such as $\lambda_{\tau e}^S \lambda_{\mu e}^S$ for electron loop and $\lambda_{\tau\tau}^S \lambda_{\mu\tau}^S$ for tauon loop. Moreover, $\tau \rightarrow e\gamma$ is also possible LFV tau decay mediated by unparticles. However this process also contains unknown, or more suppressed parameters $\lambda_{ee,\tau e,\tau\tau}^S$. These couplings may be constrained by other processes, but we neglect these here because all of them must be small or can be zero.

If there are couplings of unparticle with photons [19],

$$\frac{1}{\Lambda_U^{d_U}} \left(\lambda_\gamma F_{\mu\nu} F^{\mu\nu} + \lambda_{\tilde{\gamma}} \tilde{F}_{\mu\nu} F^{\mu\nu} \right) O_U \quad (18)$$

these operators also generate $\tau \rightarrow \mu\gamma$ at one-loop level. However, these operators are more suppressed by the factor $m_\tau/\Lambda_U \sim 10^{-3}$ than Eq. (16), and contain unknown parameters $\lambda_{\gamma(\tilde{\gamma})}$. Therefore, we again neglect these interactions.

In the next subsection, we perform numerical calculation of these LFV tau decay processes, and verify that there exist regions which do not conflict with experiments.

Numerical Calculation

Now we are ready to find whether unparticle can explain muon $g-2$ without conflicting with LFV tau decay processes. Fig. 6 show the BR of $\tau \rightarrow 3\mu$ (left) and $\tau \rightarrow \mu\gamma$ (right) as a function of d_U . In these figures, $\lambda_{\mu\tau}^S$ and $\lambda_{\mu\mu}^S$ are generated independently and randomly in the region allowed by $g-2$ experiment shown in Fig. 2 assuming $\lambda_{\mu\tau,\mu\mu}^S > 0.001$. The horizontal lines represent experimental bound Eq. (11). From the figures, one can see that some sets of $(\lambda_{\mu\tau}^S, \lambda_{\mu\mu}^S)$ give the BR below the

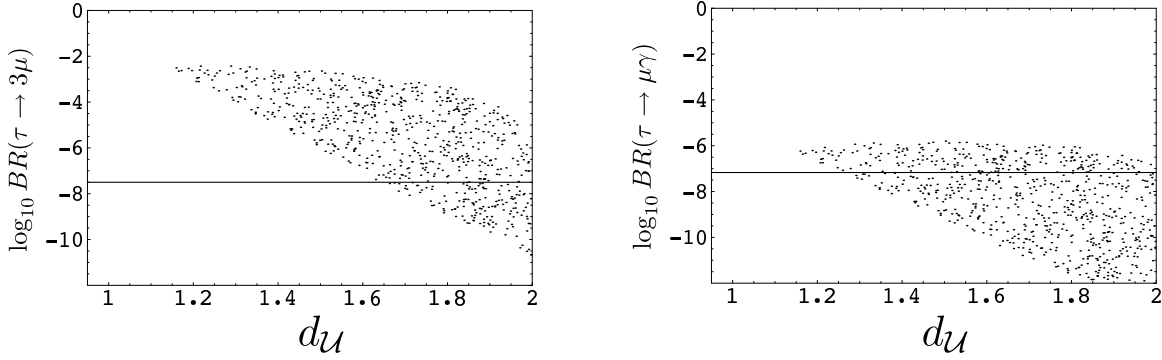


FIG. 6: Branching ratio of $\tau \rightarrow 3\mu$ (left) and $\tau \rightarrow \mu\gamma$ (right) by scalar unparticle operators with $\lambda_{\mu\mu,\mu\tau}^S > 0.001$. The horizontal line is experimental upper bound of Eq. (11). There exist solutions below the experimental bound for $d_U \gtrsim 1.6$.

experimental bound of $\tau \rightarrow 3\mu$ if $d_U \gtrsim 1.6$, while it is enough suppressed for almost all d_U for the one-loop process $\tau \rightarrow \mu\gamma$. These results are not changed for larger Λ_U , because the dependence of $\lambda^S/\Lambda_U^{d_U-1}$ is the same for all phenomena.

Fig. 7 shows the final region in the $\lambda_{\mu\tau}^S - \lambda_{\mu\mu}^S$ plane at $d_U = (1.7, 1.9)$ (left) and $d_U = (1.6, 1.8)$ (right). In the left figure, thick curves and lines correspond to $d_U = 1.7$ and thin ones to $d_U = 1.9$, and similar for the right figure. Left lower areas of each diagonal solid line representing the allowed region from $\tau \rightarrow 3\mu$ experiment Eq. (11) are superimposed on Fig. 4. Dark and light shaded areas represent the combined allowed region of all experiments for $d_U = 1.7$ (left), 1.6 (right) and $d_U = 1.9$ (left), 1.8 (right), respectively. Future experiments such as super B factory [28] will have sensitivity up to 2×10^{-10} for $\tau \rightarrow 3\mu$ and 2×10^{-9} for $\tau \rightarrow \mu\gamma$, and dotted lines are obtained from $BR(\tau \rightarrow 3\mu) < 2 \times 10^{-10}$.

Comparing the Fig. 4, the regions in which both couplings are large vanish, and those in which either of them is large remain. These regions are compatible with both muon $g - 2$ and LFV tau decay processes. This means either of the couplings has to be of $\mathcal{O}(0.1 \dots 1)$ for the case of large d_U . In the case of both the couplings are small, which is favored in the point of view of LFV tau decay, the scaling dimension d_U has to be small in order to obtain appropriate value of muon $g - 2$. Future LFV tau decay experiments will restrict the allowed regions.

The situation $\lambda_{\mu\tau}^S = 0$ is also a solution. The coupling $\lambda_{\mu\mu}^S$ can give a desired value of Δa_μ even if $\lambda_{\mu\tau}^S = 0$, and in this case LFV tau decays of our consideration can not occur.

We conclude that LFV coupling $\lambda_{\mu\tau}^S$ does not have to be zero or extremely suppressed, it can be of $\mathcal{O}(0.1 \dots 1)$, if $\lambda_{\mu\mu}^S \lesssim 10^{-2}$ and $d_U \gtrsim 1.6$.

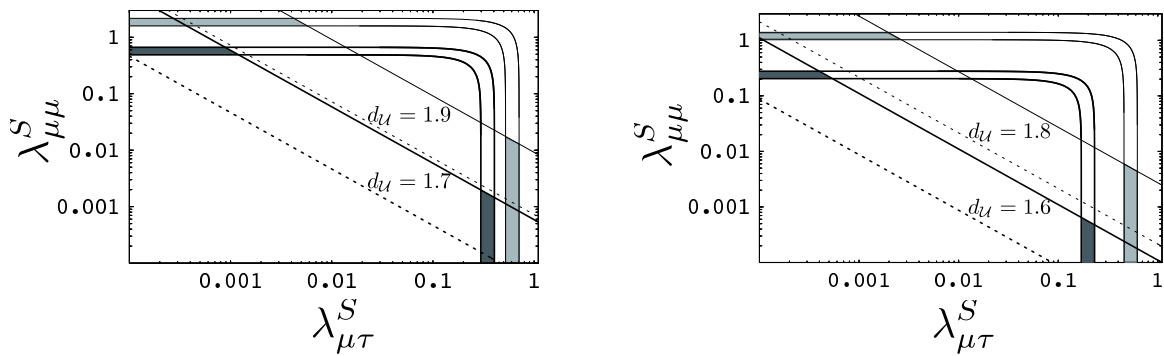


FIG. 7: Consistent region in the $\lambda_{\mu\tau}^S - \lambda_{\mu\mu}^S$ plane at $d_U = (1.7, 1.9)$ (left) and $d_U = (1.6, 1.8)$ (right). Left lower areas of each diagonal line representing the allowed region from $\tau \rightarrow 3\mu$ experiment are superimposed on Fig. 4, and dotted lines are obtained from expected future experiments. Shaded areas represent the combined allowed region of all present experiments.

V. CONCLUSIONS

We have studied muon anomalous magnetic moment and lepton flavor violating tau decay $\tau \rightarrow 3\mu$ and $\tau \rightarrow \mu\gamma$ generated by scalar unparticle interactions. Since the principle to determine unparticle interactions is still unknown, both lepton flavor violating and conserving interactions may exist, and these interactions can simultaneously generate both phenomena. We have found that scalar unparticles explain the discrepancy of experimental value of muon $g - 2$ from the Standard Model prediction, without conflicting with the experimental bound of LFV tau decay processes. When either LFV or LFC coupling vanishes, muon $g - 2$ is easily generated and LFV tau decay can not occur in any value of the scaling dimension d_U . On the other hand, when both couplings exist, these couplings and the scaling dimension are constrained by LFV tau decay. In the case of large scaling dimension ($d_U \gtrsim 1.6$), LFV coupling $\lambda_{\mu\tau}^S$ need not be small. It can be of $\mathcal{O}(1)$ if LFC coupling $\lambda_{\mu\mu}^S$ is enough small, and *vice versa*.

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